Propagation Characteristics of Striplines with Multilayered Anisotropic Media

TOSHIHIDE KITAZAWA AND YOSHIO HAYASHI

Abstract —**Various types of striplines with anisotropic media are analyzed. The analytical approach used in this paper is based on the network analytical method of electromagnetic fields, and the formulation process is straightforward for complicated structures. Some numerical results are presented and comparison is made with the results available in the literature.**

I. Introduction

THE NETWORK analytical method of electromagnetic fields has been successfully applied to analyze the propagation characteristics of planar transmission lines. The hybrid-mode analysis of single and coupled slots was presented by employing this method [1], [2]. Recently, the dispersion characteristics of single microstrip on an anisotropic substrate have been obtained using this approach [3].

Single and coupled striplines on an anisotropic substrate have been analyzed by several investigators [3]–[7], but hybrid-mode analysis is available only for the single microstrip case [3], [7], [8].

The purpose of this paper is to outline a new approach which is an extension of the treatment used in [1]–[3] and is capable of giving the propagation characteristics of various types of striplines with anisotropic media inclusively. In what follows, the formulation process is illustrated using the general structure with multilayered uniaxially anisotropic media. Two methods of solution are presented. One is based on the quasi-static approximation and it derives the transformation from the case with anisotropic layers to the case with equivalent isotropic layers. The other is based on the hybrid-mode formulation and it gives the frequency dependent solutions. The numerical results will be presented for single and coupled microstrips, coupled suspended strips, and coupled strips with overlay.

II. The Network Analytical Method of Electromagnetic Fields

Fig. 1 shows the cross section of coupled strips having multilayered uniaxially anisotropic media, whose permittivity tensors are

$$\hat{\epsilon}_i = \begin{bmatrix} \epsilon_{i\perp} & 0 & 0 \\ 0 & \epsilon_{i\perp} & 0 \\ 0 & 0 & \epsilon_{i\parallel} \end{bmatrix}, \qquad i=1,2,3. \tag{1}$$

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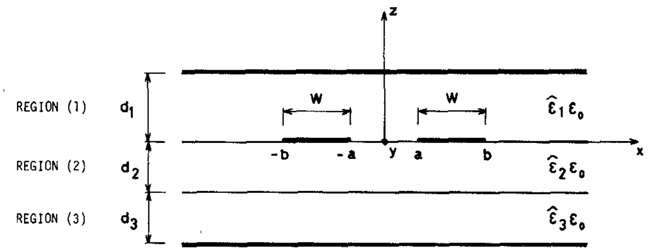

Fig. 1. General structure of coupled strips having multilayered anisotropic media.

As a first step we express the transverse fields in each region by the following Fourier integral:

$$\left.\begin{matrix} \boldsymbol{E}_t^{(i)} \\ \boldsymbol{H}_t^{(i)} \end{matrix}\right\} = \sum_{l=1}^{2} \int_{-\infty}^{\infty} \left\{ \begin{matrix} V_l^{(i)}(\alpha; z)\, \boldsymbol{f}_l(\alpha; x) \\ I_l^{(i)}(\alpha; z)\boldsymbol{z}_0 \times \boldsymbol{f}_l(\alpha; x) \end{matrix} \right\} d\alpha\, e^{-j\beta_0 y}, \qquad i=1,2,3 \tag{2}$$

where

$$\boldsymbol{f}_1 = \frac{j}{\sqrt{2\pi}} \boldsymbol{K}_0 e^{-j\alpha x}, \qquad \boldsymbol{f}_2 = \boldsymbol{f}_1 \times \boldsymbol{z}_0$$

$$\boldsymbol{K}_0 = \frac{\boldsymbol{K}}{K}$$

$$\boldsymbol{K} = \boldsymbol{x}_0 \alpha + \boldsymbol{y}_0 \beta_0, \qquad K = |\boldsymbol{K}| \tag{3}$$

where β_0 is the propagation constant in the y-direction, $\boldsymbol{x}_0$, $\boldsymbol{y}_0$, and $\boldsymbol{z}_0$ are the x-, y-, and z-directed unit vectors, respectively, and $l=1$ and $l=2$ represent E waves ($H_z=0$) and H waves ($E_z=0$), respectively. Equation (2) shows that the field components are a superposition of inhomogeneous waves whose spatial variation is $\exp\{-j(\alpha x + \beta_0 y)\}$.

Substituting the above expression into Maxwell's field equation, we obtain the following transmission-line equation in each region:

$$-\frac{d}{dz} V_l^{(i)} = j\kappa_l^{(i)} z_l^{(i)} I_l^{(i)}$$

$$-\frac{d}{dz} I_l^{(i)} = j\kappa_l^{(i)} y_l^{(i)} V_l^{(i)} \tag{4}$$

where

$$\kappa_1^{(i)} = \sqrt{\epsilon_{i\perp}\mathcal{K}_0^2 - \frac{\epsilon_{i\perp}}{\epsilon_{i\parallel}} K^2} \qquad \kappa_2^{(i)} = \sqrt{\epsilon_{i\perp}\mathcal{K}_0^2 - K^2}$$

$$z_1^{(i)} = \frac{\kappa_1^{(i)}}{\omega\epsilon_0\epsilon_{i\perp}} \qquad z_2^{(i)} = \frac{\omega\mu_0}{\kappa_2^{(i)}}$$

$$y_l^{(i)} = \frac{1}{z_l^{(i)}}, \qquad (l=1,2) \qquad \mathcal{K}_0 = \omega\sqrt{\epsilon_0\mu_0}. \tag{5}$$

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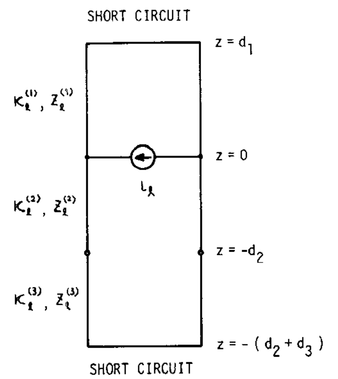

Fig. 2. Equivalent transmission-line circuits for transverse section of coupled strips.

Notice that $\kappa_1^{(i)}$ and $\kappa_2^{(i)}$ are the propagation constants in the z-direction for E waves and H waves, respectively, and $z_1^{(i)}$ and $z_2^{(i)}$ are the characteristic impedance for these waves.

The boundary conditions to be satisfied are expressed as follows:

$$V_l^{(1)}(d_1)=0 \tag{6}$$

$$V_l^{(1)}(+0)=V_l^{(2)}(-0) \tag{7a}$$

$$I_l^{(1)}(+0)-I_l^{(2)}(-0)=i_l \tag{7b}$$

$$V_l^{(2)}(-d_2+0)=V_l^{(3)}(-d_2-0) \tag{8a}$$

$$I_l^{(2)}(-d_2+0)=I_l^{(3)}(-d_2-0) \tag{8b}$$

$$V_l^{(3)}(-d_2-d_3)=0 \tag{9}$$

$$i_l=-\int_{-\infty}^{\infty} f_l^*(\alpha;x')\cdot \boldsymbol{i}(x')\,dx' \tag{10}$$

where the asterisk signifies the complex conjugate functions, and $\boldsymbol{i}(x')$ is the current density on the strip conductors at $z=0$ and may be expressed as

$$\boldsymbol{i}(x')=\boldsymbol{x}_0 i_x(x')+\boldsymbol{y}_0 i_y(x'). \tag{11}$$

Considering the transmission-line equation (4) together with the boundary conditions (6)–(9), we can obtain the equivalent circuits in the z-direction (Fig. 2). By conventional circuit theory, the mode voltages $V_l^{(i)}$ and currents $I_l^{(i)}$ in each region can be expressed in terms of i_l as

$$V_l^{(i)}(\alpha;z)=Z_l^{(i)}(\alpha;z)i_l(\alpha)$$
$$I_l^{(i)}(\alpha;z)=T_l^{(i)}(\alpha;z)i_l(\alpha). \tag{12}$$

The electromagnetic fields in each region can be obtained by substituting (12) into (2).

III. Variational Expression for the Line Capacitance

In the quasi-static approximation, the characteristic impedance and the normalized propagation constant can be obtained from the line capacitance per unit length. We will derive a variational expression of the line capacitance of the general structure shown in Fig. 1.

The longitudinal component of the electric field in region (1) can be obtained from the transverse fields according to

$$E_z^{(1)}=\frac{1}{j\omega\epsilon_0\epsilon_{1\parallel}}\nabla\cdot\left(\boldsymbol{H}_t^{(1)}\times \boldsymbol{z}_0\right). \tag{13}$$

Substituting (2) and (12) into (13) and applying $\beta_0\to 0$, $E_z^{(1)}$ can be obtained as

$$E_z^{(1)}(x,z)=\frac{1}{2\pi}\cdot\frac{1}{\omega\epsilon_0\epsilon_{1\parallel}}\cdot\iint_{-\infty}^{\infty}\alpha T_1^{(1)}(\alpha;z)i_x(x')\cdot e^{-j\alpha(x-x')}\,dx'\,d\alpha. \tag{14}$$

Performing the integration by parts, using the equation of continuity

$$-j\omega\sigma(x')=\frac{d}{dx'}i_x(x') \tag{15}$$

and applying the zero frequency approximation $\omega\to 0$ to (14), we get

$$E_z^{(1)}(x,z)=\frac{1}{2\pi\epsilon_0}\iint_{-\infty}^{\infty}F(\alpha)p_1\frac{\cosh\{p_1(z-d_1)|\alpha|\}}{\sinh(p_1 d_1|\alpha|)}\cdot\sigma(x')e^{-j\alpha(x-x')}\,d\alpha\,dx' \tag{16}$$

where

$$F(\alpha)=\frac{1}{\epsilon_{1e}\coth(p_1d_1|\alpha|)+\epsilon_{2e}L} \tag{17}$$

$$L=\frac{1+\frac{\epsilon_{2e}}{\epsilon_{3e}}\tanh(p_2d_2|\alpha|)\tanh(p_3d_3|\alpha|)}{\tanh(p_2d_2|\alpha|)+\frac{\epsilon_{2e}}{\epsilon_{3e}}\tanh(p_3d_3|\alpha|)} \tag{18}$$

$$p_i=\sqrt{\frac{\epsilon_{i\perp}}{\epsilon_{i\parallel}}},\quad \epsilon_{ie}=\sqrt{\epsilon_{i\parallel}\epsilon_{i\perp}} \tag{19}$$

and $\sigma(x')$ is the charge distribution on the strip conductors. The potential distribution at $z=0$ becomes

$$V(x)=\int_0^{d_1}E_z(x,z)\,dz$$
$$=\int_a^b\int_0^{\infty}G(\alpha;x|x')\sigma(x')\,d\alpha\,dx' \tag{20}$$

where

$$G(\alpha;x|x')=\frac{2}{\pi\epsilon_0}\cdot\frac{F(\alpha)}{|\alpha|}\cos\alpha x\cos\alpha x'\ \text{(for even modes)}$$
$$=\frac{2}{\pi\epsilon_0}\cdot\frac{F(\alpha)}{|\alpha|}\sin\alpha x\sin\alpha x'\ \text{(for odd modes)}. \tag{21}$$

On the strip conductor $a<x<b$, $V(x)$ is equal to a constant V_0, that is, the potential difference between the strip and the ground conductors

$$V(x)=V_0=\int_a^b\int_0^{\infty}G(\alpha;x|x')\sigma(x')\,d\alpha\,dx',\qquad a<x<b. \tag{22}$$

From (22), the variational expression for the line capacitance can be obtained [9]

$$\frac{1}{C}=\frac{V_0}{Q}=\frac{\iint_a^b\int_0^\infty \sigma(x)G(\alpha;x|x')\sigma(x')\,d\alpha\,dx'\,dx}{\left\{\int_a^b\sigma(x)\,dx\right\}^2} \tag{23}$$

where Q is the total charge on the strip conductor $a<x<b$

$$Q=\int_a^b\sigma(x)\,dx. \tag{24}$$

Equation (23), together with (21) and (17)–(19), suggests that, in the quasi-static approximation, coupled strips with multilayered uniaxially anisotropic media can be transformed into the case with effective isotropic layers, of which the effective thickness and the relative permittivity are $\sqrt{\epsilon_{i\perp}/\epsilon_{i\parallel}}\cdot d_i$ and $\sqrt{\epsilon_{i\perp}\epsilon_{i\parallel}}$, respectively.

IV. Hybrid-Mode Analysis

The analytical method for the frequency-dependent characteristics of coupled strips shown in Fig. 1 is explained here. This method is analogous to those used in [1]–[3] and will be outlined briefly.

The transverse electric fields, which were obtained in the integral representation in Section II, must be zero on the strip conductors at $z=0$. This gives the integral equation on the current density $\boldsymbol{i}(x)$ and the propagation constant in the y-direction β_0. The unknown current densities $i_x(x)$ and $i_y(x)$ are expanded in terms of known sets of basis functions as follows:

$$i_x(x)=\sum_{k=1}^{N_x}a_{xk}f_{xk}(x)$$

$$i_y(x)=\sum_{k=1}^{N_y}a_{yk}f_{yk}(x) \tag{25}$$

where a_{xk} and a_{yk} are unknown coefficients. Substituting (25) into the integral equation and applying Galerkin's procedure, we obtain a set of simultaneous equations on the unknown a_{xk} and a_{yk}. The propagation constant can be obtained by searching the nontrivial solution.

The definition for the characteristic impedance is not uniquely specified due to the propagation of the hybrid mode. The definition chosen here is

$$Z_0=\frac{P_{\text{ave}}}{I_0^2} \tag{26}$$

where I_0 is the total current on one strip conductor, and P_{ave} is the average power flow along the y-direction.

V. Basis Functions

The line capacitance is calculated by applying the Ritz procedure to the variational expression (23). In this procedure, we express the unknown charge distribution $\sigma(x)$ as

$$\sigma(x)=f_0(x)+\sum_{k=1}^{N}A_kf_k(x) \tag{27}$$

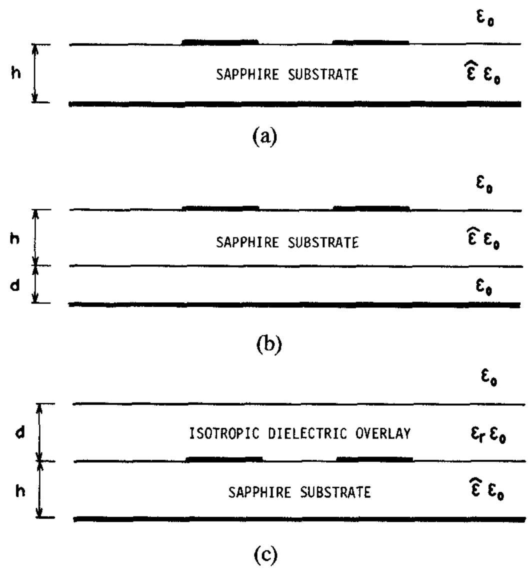

Fig. 3. (a) Coupled microstrips. (b) Coupled suspended strips. (c) Coupled strips with overlay.

where A_k are variational parameters which are determined so that the best approximation is obtained.

In the numerical computations, the choice of the basis functions, $f_k(x)$ in (27) and $f_{xk}(x)$ and $f_{yk}(x)$ in (25), is important. It is desirable that the edge effect should be properly accounted for, and that the approximation to the true value should be systematically improved by increasing the number of basis functions. Taking these requirements into account, we adopt the following families of functions for basis functions:

$$f_{xk}(x)=U_k\left\{\frac{2(x-S)}{W}\right\}$$

$$\left.\begin{matrix}f_{k-1}(x)\\ f_{yk}(x)\end{matrix}\right\}=\frac{T_{k-1}\left\{\frac{2(x-S)}{W}\right\}}{\sqrt{1-\left\{\frac{2(x-S)}{W}\right\}^2}}$$

$$S=(a+b)/2,\quad W=b-a \tag{28}$$

where $T_k(y)$ and $U_k(y)$ are Chebyshev's polynomials of the first and second kind, respectively. By the use of these basis functions, the fast convergence to the exact values is obtained. Preliminary computations show that $N=2$ in (27) and $N_x=N_y=2$ in (25) are sufficient for any case.

VI. Numerical Results

Numerical computations were carried out for single and coupled microstrips (Fig. 3(a)), coupled suspended strips (Fig. 3(b)), and coupled strips with overlay (Fig. 3(c)). In the open microstrip configurations of Fig. 3, the boundary condition (6) or (9) for Fig. 1 should be replaced by the radiation condition. However, the resulting equations thus obtained are the same as those for Fig. 1 in which $d_1\to\infty$ or $d_3\to\infty$. These calculations were performed using the same computer program with very little modification.

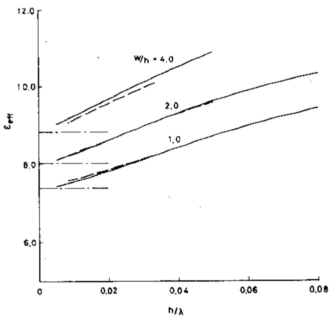

Fig. 4. Dispersion characteristics of single microstrip on sapphire. ($\epsilon_{\perp} = 9.4$, $\epsilon_{\parallel} = 11.6$; —hybrid-mode; ----quasi static; — — — El-Sherbiny's [4].)

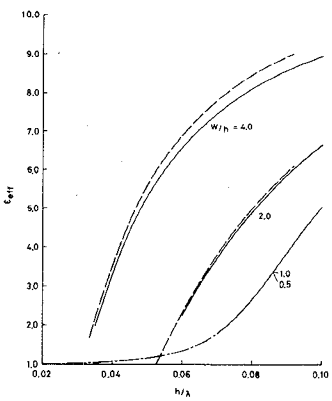

Fig. 5. Dispersion characteristics of the first higher order mode of single microstrip on sapphire. (—this theory; -·-—TM_0 mode of a sapphire-coated conductor; — — —El-Sherbiny's [4].)

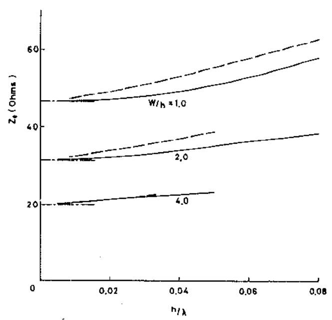

Fig. 6. Characteristic impedance of single microstrip on sapphire.

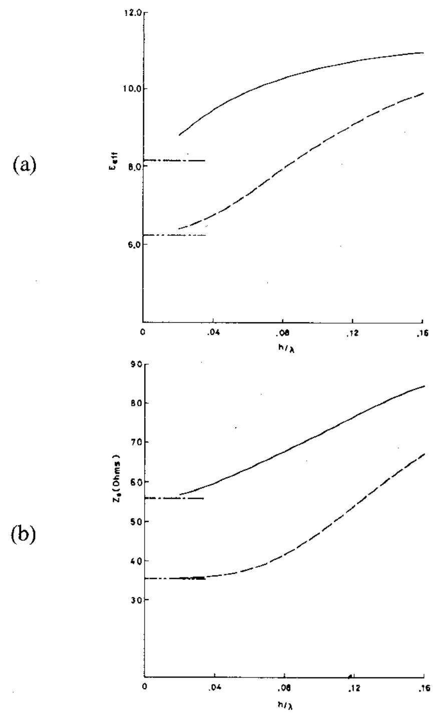

Fig. 7. (a) Dispersion characteristics of coupled microstrips on sapphire. (b) Characteristic impedance of coupled microstrips on sapphire. ($W/h = 1$, $a/h = 0.25$; —even mode (hybrid-mode); — — —odd mode (hybrid-mode); -·-—even mode (quasi-static); -··-—odd mode (quasi-static).)

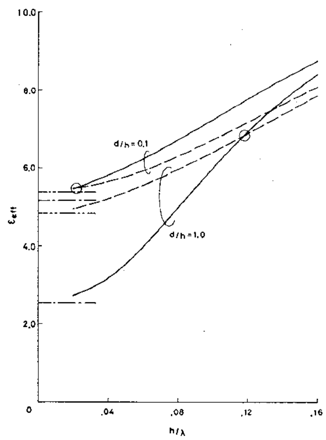

Fig. 8. Dispersion characteristics of coupled suspended strips. ($\epsilon_{\perp} = 9.4$, $\epsilon_{\parallel} = 11.6$, $W/h = 1$, $a/h = 0.25$; —even mode (hybrid-mode); — — —odd mode (hybrid-mode); -·-— even mode (quasi-static); -··-— odd mode (quasi-static).)

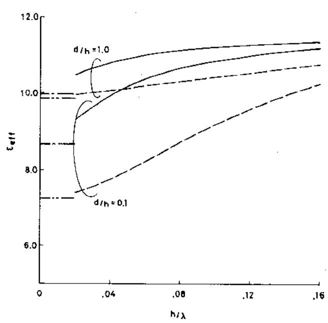

Fig. 9. Dispersion characteristics of coupled strips with overlay. ($\epsilon_{\perp} = 9.4$, $\epsilon_{\parallel} = 11.6$, $\epsilon_r = 9.6$, $W/h = 1$, $a/h = 0.25$; —even mode (hybrid-mode); — — —odd mode (hybrid mode); -·-— even mode (quasi-static); -··-—odd mode (quasi-static).)

Fig. 4 shows the dispersion characteristics, the frequency dependence of the effective dielectric constant $\epsilon_{eff} = \beta_0^2/\omega^2\epsilon_0\mu_0$, of single microstrip on sapphire substrates, where ϵ_{eff} for the dominant mode is reported and compared with the results of El-Sherbiny [8]. The agreement is quite good, although some disagreement appears for wide strips.

Fig. 5 shows the dispersion characteristics of the first higher order mode, which are also compared with those from [8]. Fig. 5 also presents the dispersion characteristics of the TM_0 surface wave of the sapphire coated conductor which results when $W = 0$. When the strip is not so wide compared with the substrate, the dispersion characteristics of the first higher order mode are indistinguishable from those of the TM_0 surface wave.

The frequency dependence of the characteristic impedance of single microstrip is shown in Fig. 6. Comparison of the results by this method and those from [8] shows that both results converge to the quasi-static values calculated from (23), but that some discrepancies appear at high frequencies. For single microstrip, the characteristic impedance is defined as

$$Z_0 = \frac{2P_{\text{ave}}}{I_0^2} \tag{29}$$

instead of (26) in our calculations, whereas it is defined as the ratio of the voltage at the center of the strip to the total longitudinal current in [8].

The dispersion characteristics of coupled microstrips, coupled suspended strips, and coupled strips with a dielectric overlay are depicted in Figs. 7, 8, and 9, respectively. It should be noted that the dispersion characteristics of the even mode of coupled suspended strips is more sensitive than that of the odd mode to the variation in d/h, therefore the frequency at which both modes have the equal phase velocity varies largely.

VII. Conclusions

Various types of striplines with anisotropic media have been analyzed using the same approach, which is based on the network analytical method of electromagnetic fields. In this analytical approach, the derivation of Green's functions is based on the conventional circuit theory, therefore the formulation for the complicated structures is straightforward.

Computations have been carried out by employing the efficient method based on the Ritz and Galerkin procedure to calculate the propagation characteristics of single and coupled microstrips, coupled suspended strips, and coupled strips with overlay. Numerical results of single microstrip were compared with other available data.

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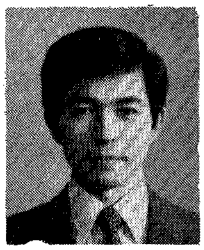

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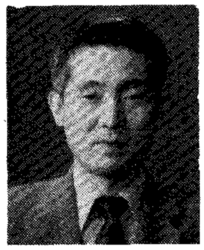

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